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Supersymmetric D-branes on $\mathrm{SU}(2)$ structure manifolds

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Abstract: We employ generalized complex geometry to investigate supersymmetric embeddings of D-brane probes in a large class of SU(2) structure manifolds. This class includes the gravity dual of mass deformation and marginal beta deformation of $\mathcal{N} = 4$ SYM gauge theory. We find supersymmetric configurations of D-branes with different dimensionality and propose their interpretation in the dual gauge theory.

Keywords: D-branes, AdS-CFT Correspondence, Brane Dynamics in Gauge Theories.

Contents

1. Introduction

Strings and supergravity backgrounds with non trivial RR and NS fluxes are intensively studied in the AdS/CFT correspondence [1] and in string compactification (see [2] and reference therein), in order to find string models holographically dual to more realistic gauge theories or to obtain sensible phenomenology from compactification. Here D-branes are successfully used as probes to explore the geometric properties of known backgrounds, and to provide further insights in the gauge/gravity duality. We focus on type IIB supergravity solutions which preserve four dimensional Poincaré invariance and $\mathcal{N} = 1$ supersymmetry. They correspond to a warped product of the four dimensional Minkowski spacetime and an internal six dimensional manifold $\mathcal{M}$, which can support fluxes. In the presence of non trivial background fluxes, the back-reacted internal manifold $\mathcal{M}$ is no longer Calabi Yau. There are special classes of solutions [3] where the internal manifold is conformal Calabi Yau, but in general [4, 5] the internal manifold with fluxes can be far different from the

Calabi Yau case. The formalism of G-structures [6] and Generalized Complex Geometry (GCG) [7, 8, 4, 5] provide powerful tools to describe such manifolds. In GCG the basic objects are pure spinors, formal sums of even and odd forms. Their existence imposes topological constraints on the tangent and cotangent bundles of the internal manifold. Supersymmetry requires that the internal manifold has a SU(3) $\times$ SU(3) structure on $T_M \oplus T_M^*$, which may be further restricted to SU(3) or SU(2) structures on T_M. The SU(3) structure has been much studied, e.g. [9], while the SU(2) case has been explored in [10] and, using GCG, in [11]. As a matter of fact, supergravity solutions with fluxes dual to massive and marginal deformations of superconformal gauge theories are expected to be described by SU(2) structure manifolds. Such manifolds are characterized by the existence of a globally defined nowhere vanishing vector field.

In the GCG language the preservation of $\mathcal{N} = 1$ supersymmetry is achieved by imposing a pair of differential equations for the pure spinors. The authors of [11] made an ansatz for pure spinors of SU(2) structure manifolds and performed a detailed analysis of these pair of supersymmetry equations. Their ansatz covers a large class of solutions. In particular the Pilch Warner [12] and the Lunin Maldacena [13] ones are included: they are the gravity duals of the single mass deformation and of the beta marginal deformation of $\mathcal{N} = 4$ SYM, respectively.

In the GCG framework the supersymmetry conditions for D-branes probing SU(3) $\times$ SU(3) backgrounds have been established in [14, 15] (see also [16]). They are a set of constraints on the pull back of the pure spinors on the world volume of the D-brane. In [15] the supersymmetry conditions were given for D-branes filling Minkowski space time (space time filling), filling three space time directions (domain walls) and two space time directions (effective strings).

The addition of D-brane probes to the class of solutions of [11] can provide other interesting tests of the AdS/CFT correspondence. Supersymmetric configurations of D-branes can identify the moduli space of vacua of the dual gauge theory, in both the abelian and the non abelian branches. $D5$ domain wall like configurations can lead in the dual description to three dimensional defects, interacting with the conformal four dimensional gauge theory; the defect gauge invariant operators can then be mapped into the Kaluza Klein modes of the wrapped brane [17]. The addition of space time filling $D7$-branes corresponds to adding massless or massive flavours [18] and their fluctuations give the meson spectrum of the dual flavoured gauge theory.

In [11] the space time filling $D3$-brane configurations have been analyzed and it was shown that the supersymmetry conditions for such branes reproduce the mesonic moduli space of vacua of the dual field theory. Moreover the $D5$-brane configuration with world volume flux, related to the non abelian phase of the beta deformed gauge theory [13, 19], was recovered.

In this paper we investigate new supersymmetric D-brane configurations in the class of SU(2) structure manifolds of [11], and we propose the dual gauge theory interpretation as well as possible applications of the results.

We look for supersymmetric $D5$ domain wall like configurations finding a supersymmetric embedding which can be used to holographically study three dimensional defects coupled to the massive deformation of $\mathcal{N} = 4$ SYM.

We study a supersymmetric embedding of space time filling $D5$-branes with non trivial world volume flux in the Pilch Warner solution.

We explore different $D7$ supersymmetric embeddings suitable for adding flavour to the whole class of solutions, suggesting in each case the dual flavored gauge theory. These embeddings identify supersymmetric four cycles. Although the formalism we adopt does not apply to the non static case, these supersymmetric four cycles should be mapped, with a strategy similar to [20], to non static configurations of $D3$ branes (giant gravitons) in this class of backgrounds.[1]

Finally, we find supersymmetric configurations of $D3$ and $D7$ branes which behave as effective strings in the four dimensional gauge theory description.

The paper is organized as follows. In section 2 we outline the spinor ansatz for SU(2) structure manifolds [11] and in section 3 the GCG supersymmetry conditions for D-branes [15]. In section 4, after a brief survey of the supersymmetric family of backgrounds which includes the PW flow, we look for supersymmetric embeddings of D-branes. We present different D-brane configurations and we solve their supersymmetry conditions, identifying supersymmetric embeddings. We give some details on the computations and we interpret the supersymmetric configurations in the dual gauge theory. The same analysis is carried out for D-brane probes in the LM geometry in section 5. In the appendices we recall some useful definitions.

2. SU(2) structure manifolds and pure spinors

The ten dimensional metric is

$$ds^2_{10} = e^{2A}\eta_{\mu\nu}dx^\mu dx^\nu + ds^2_6 \tag{2.1}$$

where the warp factor A is a function of the internal coordinates. The internal six dimensional manifold has SU(2) structure. An SU(2) structure is characterized by two nowhere vanishing spinors which are never parallel

$$\eta_+ \qquad \chi_+ = \frac{1}{2}z\cdot\eta_- \tag{2.2}$$

where η_- is the complex conjugate of η_+ and we denote with $\cdot$ the Clifford multiplication $z_m\gamma^m$. The six dimensional chiral spinors $\eta^i_\pm$, which are the supersymmetry parameters, are then constructed

$$\eta^1_+ = a\eta_+ + b\chi_+ \qquad \eta^2_+ = x\eta_+ + y\chi_+ \tag{2.3}$$

with a, b, x, y complex functions of the internal coordinates. The ten dimensional supersymmetry parameters can be written as

$$\epsilon_1 = \zeta_+\otimes\eta^1_+ + \zeta_-\otimes\eta^1_- \tag{2.4}$$
$$\epsilon_2 = \zeta_+\otimes\eta^2_+ + \zeta_-\otimes\eta^2_- \tag{2.5}$$

[1] For giants in the beta deformed background see [21].

where $\zeta_\pm$ are four dimensional chiral spinors. Given the never vanishing spinors just introduced, a SU(2) structure manifold admits the following globally defined forms built as bilinears in the spinors

$$j = \frac{i}{2}\chi_+^\dagger\gamma_{mn}\chi_+ dx^m \wedge dx^n - \frac{i}{2}\eta_+^\dagger\gamma_{mn}\eta_+ dx^m \wedge dx^n \tag{2.6}$$

$$\omega = -i\chi_+^\dagger\gamma_{mn}\eta_+ dx^m \wedge dx^n \tag{2.7}$$

$$z = -2\chi_-^\dagger\gamma_m\eta_+ dx^m \tag{2.8}$$

where z is a complex 1-form, j a real 2-form, and ω a (2,0)-form satisfying

$$\omega \wedge j = 0 \qquad j \wedge j = \frac{1}{2}\omega\wedge\bar\omega \qquad z\llcorner j = z\llcorner\omega = 0 \tag{2.9}$$

The 1-form z is the globally defined complex vector characterizing the SU(2) structure.

In GCG the relevant equations can be written in terms of poliforms with definite parity, the pure spinors. They are bispinors built by tensoring the supersymmetry parameters of the internal manifold

$$\Phi_1 = \eta_+^1 \otimes \eta_+^{2\dagger} \tag{2.10}$$

$$\Phi_2 = \eta_+^1 \otimes \eta_-^{2\dagger} \tag{2.11}$$

and are annihilated by six combinations of Clifford(6,6) gamma matrices. From (2.3) they read

$$\begin{aligned}\Phi_1 &= \frac{1}{8}[a\bar{x}e^{-ij} + b\bar{y}e^{ij} - i(a\bar{y}\omega + \bar{x}b\bar\omega)] \wedge e^{z\wedge\bar{z}/2} \\ \Phi_2 &= \frac{1}{8}[i(by\bar\omega - ax\omega) + (bxe^{ij} - aye^{-ij})] \wedge z\end{aligned} \tag{2.12}$$

The SU(3) structure case is for $b = 0 = y$.

The ansatz used in [11] for the six dimensional supersymmetry parameters is the following

$$\eta_+^1 = a\eta_+ + b\chi_+ \qquad \eta_+^2 = -i(a\eta_+ - b\chi_+) \tag{2.13}$$

where the functions of (2.3) are parametrized as

$$a = ix = ie^{A/2}\cos\phi\, e^{i\alpha} \qquad b = -iy = -ie^{A/2}\sin\phi\, e^{i\beta} \tag{2.14}$$

Here $\cos\phi$, $\sin\phi$, α and β are functions of the internal coordinates. The two supersymmetry parameters η_+^1, η_+^2 can be brought to the form (2.13) if and only if $\mathrm{Re}(a\bar{x} + b\bar{y}) = 0$. This corresponds to admit a non trivial mesonic moduli space of vacua [11].

We are interested in D-branes probing the class of backgrounds specified by the ansatz (2.13), (2.14). This contains a family of supersymmetric backgrounds with constant dilaton (which itself includes the PW flow), and the gravity dual of beta deformation. Since the norms of the spinors η_1 and η_2 are equal, supersymmetric D-branes are admitted [15].

JHEP09(2007)123

	θ	a	b
STF	0	1	2
DW	θ_{DW}	2	1
ES	$-\frac{\pi}{2}$	1	2

Table 1: Summary of the supersymmetry conditions.

3. Supersymmetry conditions for probe D-branes

In GCG the main tool to analyze supersymmetric embeddings of D-branes is the generalized calibration introduced in [14, 15]. We will consider space time filling branes (STF), domain walls (DW) and effective strings (ES) wrapping a submanifold Σ of the internal manifold. The supersymmetry conditions for these extended objects in terms of the pure spinors and their projection on the world volume read[2]

$$P_\Sigma[\mathrm{Im}(ie^{i\theta}\Phi_a)] \wedge e^{\mathcal{F}} = 0 \tag{3.1}$$

$$P_\Sigma[(i_n + g_{nm}dx^m\wedge)\Phi_b] \wedge e^{\mathcal{F}} = 0 \qquad a,b = 1,2 \tag{3.2}$$

where g_{nm} is the internal metric, i_n and $dx^m\wedge$ are the usual operators mapping a p form in a $p-1$ and $p+1$ form respectively, and finally[3] $\mathcal{F} = F - P_\Sigma[B]$, where F is the world volume flux. The pullback on the world volume of the D-brane is denoted by P_Σ. Space time filling branes, domain walls and effective strings are summarized in table 1, where θ_{DW} is an arbitrary constant [15].

The same dictionary of [15] is used to label the possible embeddings. However, since the internal manifold is non compact, we should distinguish between the cases when the wrapped submanifold Σ is itself compact or non compact. We will comment on this point where needed.

4. D-branes on the family of supersymmetric backgrounds

4.1 The family of supersymmetric backgrounds

We now briefly review the family of supersymmetric backgrounds analyzed in [11] which includes the PW flow [12]. The PW solution is the gravity dual of the massive deformation of $\mathcal{N} = 4$ SYM

$$W = h\mathrm{Tr}\Phi_3[\Phi_1, \Phi_2] + m\mathrm{Tr}\Phi_3^2 \tag{4.1}$$

which flows in the IR to a non trivial fixed point [22]. The gravity dual is asymptotically AdS in the UV and warped AdS in the IR. It is included in the following more general ansatz [11] which is a family of supersymmetric backgrounds

$$ds_6^2 = e^{-2A}\left(\eta_i A_{i\bar{j}}\bar{\eta}_{\bar{j}} + z\bar{z}\right) \qquad i,j = 1,2 \tag{4.2}$$

[2]We do not consider the orientation conditions on these objects.

[3]We are using the conventions of [4, 5, 11] which differs for an H_{NS} sign with [15].

JHEP09(2007)123

where z is the globally defined vector characterizing the SU(2) structure. The matrix $A_{i\bar{j}}$ is hermitian, and the vielbeins are defined in terms of local complex coordinates z_i

$$z_1 = \rho_1 + i\sigma_1 \qquad z_2 = \rho_2 + i\sigma_2 \qquad z_3 = \log u + i\sigma_3 \tag{4.3}$$

$$\eta_1 = dz_1 + \alpha_1 dz_3 \qquad \eta_2 = dz_2 + \alpha_2 dz_3 \qquad z = \sqrt{a_3}\, u dz_3 \tag{4.4}$$

with a_3 real and α_i complex functions of z_i. The globally defined two forms are

$$j = \frac{i}{2} A_{i\bar{j}}\, \eta_i \wedge \eta_{\bar{j}} \tag{4.5}$$

$$\omega = i\sqrt{\det A}\, \eta_1 \wedge \eta_2 \tag{4.6}$$

There are also non trivial RR and NS fluxes

$$*F_5 = -e^{-4A} d(e^{4A} \cos 2\phi) \tag{4.7}$$

$$C_2 = \mathrm{Re}\left[\frac{2ie^{i(\alpha-\beta)}\sqrt{\det A}}{e^{2A}\sin 2\phi}(dz_1 \wedge dz_2 - \sin^2\phi\, \eta_1 \wedge \eta_2)\right] \tag{4.8}$$

$$B_2 = -\mathrm{Im}\left[\frac{2ie^{i(\alpha-\beta)}\sqrt{\det A}}{e^{2A}\sin 2\phi}(dz_1 \wedge dz_2 - \sin^2\phi\, \eta_1 \wedge \eta_2)\right] \tag{4.9}$$

The dilaton is constant, parametrising the RG line of dual conformal gauge theories.

The supersymmetry equations for this background [11] imply that $\alpha = \frac{1}{2}(\sigma_1+\sigma_2+3\sigma_3)$, $\beta = -\frac{1}{2}(\sigma_1 + \sigma_2 - \sigma_3)$ and that the functions $a_3, \alpha_i, A_{i\bar{j}}$ can be obtained as derivatives of a single function $F(z_i, \bar{z}_{\bar{j}})$. These are all real for the subclass of this family of backgrounds which have an $\mathrm{U}(1)^3$ symmetry, i.e. when the function $F(z_i, \bar{z}_{\bar{j}})$ does not depend on the phases σ_i. We call this the *toric subclass*; the PW flow belongs to it.

The detailed expressions for the family of backgrounds and how to recover the PW flow are reported in the appendix A.

The pure spinors (2.12) are constructed with the rescaled forms $z \to e^{-A}z$ and $(j,\omega) \to (e^{-2A}j, e^{-2A}\omega)$ which refer to the complete six dimensional metric (4.2).

We look for supersymmetric embeddings of Dp-branes (with world volume coordinates ξ_a $(a = 0, \ldots, p)$) in this family of supersymmetric backgrounds, allowing in one case for non trivial world volume gauge flux. The main tools are the conditions (3.1), (3.2).

Even if the family of backgrounds is larger, we shall take the PW solution as a paradigm for the gauge theory dual interpretation of the brane configurations.

4.2 $D5$ domain walls

We study now a supersymmetric D-brane probe placed at $x_3 = 0$ and which fills three space time dimensions $(\xi_0, \xi_1, \xi_2) = (x_0, x_1, x_2)$. It can be viewed as a domain wall solution separating supersymmetric vacua. However, when the wrapped cycle is non compact, the domain wall interpretation would imply an infinite potential barrier. Instead in the AdS/CFT interpretation it is a three dimensional defect coupled to the four dimensional dual gauge theory.

In the $AdS_5 \times S^5$ case there are non trivial supersymmetric embeddings where a $D5$-brane wraps an AdS_4 inside the AdS_5 plus a trivial 2-sphere inside the S^5 [23]. The $D5$

JHEP09(2007)123

brane should shrink around this 2-sphere but the correspondent tachionic mode does not lead to instability because its mass is above the BF bound [24]. This configuration has been studied in [17] as a three dimensional defect in $\mathcal{N} = 4$ SYM.

We look for similar configurations of $D5$-brane in the family of supersymmetric backgrounds of section 4.1. We attempt the following three cycle embedding

$$z_k = e^{i\tau_k}(\xi_{k+2} + ic_k) \qquad \bar{z}_k = e^{-i\tau_k}(\xi_{k+2} - ic_k) \qquad k = 1, \ldots, 3 \tag{4.10}$$

with τ_k and c_k constants, and with no world volume flux, $F = 0$. This ansatz covers for example the real slice ($\tau_k = 0$, $\forall k$) and the imaginary slice ($\tau_k = \frac{\pi}{2}$, $\forall k$).

We restrict ourselves to the *toric subclass*. The complex functions $\alpha_i, A_{i\bar{j}}$ characterizing the metric are then real and the computations simplify. We compute the supersymmetry conditions (3.1) and (3.2) in the DW case of table 1.

The supersymmetry condition (3.1) results

$$P_\Sigma[\mathrm{Im}(ie^{i\theta_{DW}}\Phi_2)] \wedge e^{\mathcal{F}} = \frac{1}{8}\mathrm{Im}[e^{-2A}\sqrt{a_3}u\sqrt{\det A}\, e^{i(\theta_{DW}+2\beta-\tau_1-\tau_2+\tau_3)}]\, d\xi_3 \wedge d\xi_4 \wedge d\xi_5 \tag{4.11}$$

where the functions are intended evaluated on the world volume. A choice of the constant phase θ_{DW} can make it vanish only if the phase factor β does not depend on the embedding coordinates ξ_{k+2}. This can be achieved taking the real slice ($\tau_k = 0$, $\forall k$), such that $\beta = -\frac{1}{2}(c_1 + c_2 - c_3)$. Then we choose $\theta_{DW} = -2\beta$ and the expression (4.11) vanishes.

For the real slice ($\tau_k = 0$, $\forall k$), a detailed analysis shows that the supersymmetry conditions (3.2) are satisfied provided $\alpha = \beta + \frac{\pi}{2}$. This implies the following relation between the constants c_k

$$c_1 + c_2 + c_3 = \frac{\pi}{2} \tag{4.12}$$

Hence we conclude that for the *toric subclass* a $D5$ brane embedded as in (4.10) with $\tau_k = 0$, with the constants c_k satisfying (4.12) and with $\theta_{DW} = (c_1 + c_2 - c_3)$ is supersymmetric. In particular, such $D5$ brane is supersymmetric in the PW flow, since it belongs to the *toric subclass*. In the PW geometry (see the appendix A) the $D5$ brane fills the three radial directions.

This embedding can be used to study three dimensional defects in the massive deformation of $\mathcal{N} = 4$. The c_i give the distance between the supersymmetric $D5$-brane and the D-branes which generate the background. They represent masses for the 3D hypermultiplet of the defect theory.

4.3 Spacetime filling D-branes

In this section we study D-brane probes filling all the Minkowski directions $\xi_\mu = x_\mu$ ($\mu = 0, \ldots, 3$). The supersymmetry conditions are (3.1) and (3.2) in the STF case of table 1. We analyze here supersymmetric $D5$-brane embeddings with world volume flux, and $D7$ flavour branes.

JHEP09(2007)123

4.3.1 $D5$-branes

We take the following two cycle embedding Σ for a $D5$ brane probing the background of section 4.1

$$z_k = e^{i\tau_k}(\xi_{k+3} + ic_k) \qquad k = 1, 2 \qquad z_3 = c_3 + ic_4 \tag{4.13}$$

with c_k and τ_k real constants. We allow for a generic world volume flux F. The only non trivial supersymmetry conditions for this configuration are the (3.1) and the z component of (3.2), since $\Phi_2 = \cdots \wedge z$ and $P_\Sigma[z] = 0$ from (4.13). The first one reads

$$P_\Sigma[\mathrm{Im}(i\Phi_1)] \wedge e^{\mathcal{F}} = -\frac{ie^{-A}}{16}(A_{1\bar{2}}e^{i(\tau_1-\tau_2)} - A_{2\bar{1}}e^{-i(\tau_1-\tau_2)})\, d\xi_4 \wedge d\xi_5 \tag{4.14}$$

and does not depend on the two form flux $\mathcal{F} = F - P[B]$ since $P_\Sigma[\mathrm{Im}(i\Phi_1)]|_0 = 0$. This expression cannot be made vanishing in general by a simple choice of the phases τ_1, τ_2. However, if we restrict ourselves to the *toric subclass* the matrix $A_{i\bar{j}}$ is real and symmetric, and $A_{1\bar{2}} = A_{2\bar{1}}$. If we then choose $\tau_1 = \tau_2$ the expression (4.14) vanishes.

We compute the z component of the second supersymmetry condition

$$P_\Sigma[(i_z + g_{z\bar{z}}\bar{z}\wedge)\Phi_2] \wedge e^{\mathcal{F}} = -\frac{ie^{-2A}}{8}(F_{\xi_4\xi_5}e^{2A}e^{i(\alpha+\beta)}\sin 2\phi + \sqrt{\det A}e^{-i(\tau_1+\tau_2-2\beta)})\, d\xi_4 \wedge d\xi_5 \tag{4.15}$$

where $F_{\xi_4\xi_5}$ is the world volume flux. The expression (4.15) vanishes if we turn on

$$F = -e^{-i(\tau_1+\tau_2+\alpha-\beta)}\frac{\sqrt{\det A}}{e^{2A}\sin 2\phi}d\xi_4 \wedge d\xi_5 \tag{4.16}$$

which for consistency should be real. The choices

$$\tau_1 = \tau_2 = 0 \qquad \alpha - \beta = c_1 + c_2 + c_3 = 0 \tag{4.17}$$

make the flux (4.16) real, since the phase factor in (4.16) is now independent of the embedding coordinates ξ_{k+3} and moreover it vanishes. We conclude that the choices (4.16) and (4.17) make the $D5$ brane configuration (4.13) supersymmetric in the *toric subclass.*

However particular care is needed in considering this embedding; indeed we observe that the $D5$ brane wraps a non compact submanifold and then the flux F is along non compact coordinates (see for example the coordinates for the PW geometry in appendix A).

4.3.2 $D7$ flavour branes

Here we look for supersymmetric $D7$-brane embeddings suitable for adding flavours to the family of backgrounds of section 4.1. The $D7$ branes should wrap a non compact four cycle in order to make the flavour symmetry group global. Adding N_f $D7$ branes on this non compact four cycle is dual to add N_f flavours with symmetry group $\mathrm{SU}(N_f)$ to the $\mathrm{SU}(N_c)$ gauge theory provided $N_f < N_c$, so that the back-reaction of the $D7$-branes can be neglected. The shape of the $D7$ supersymmetric embedding sets the interaction terms in the superpotential between the flavours and the chiral superfields of the dual gauge theory as well as possible masses for the flavours.

In a SU(2) structure manifold the globally defined vector z naturally identifies a four dimensional submanifold Σ where $P_\Sigma[z] = 0$. Thus we attempt the embedding with $P_\Sigma[z] = 0$, i.e. we place $D7$ branes as

$$\begin{aligned} x_\mu &= \xi_\mu \qquad\qquad \mu = 0, \dots, 3 \\ z_k &= \xi_{k+3} + i\xi_{k+5} \qquad k = 1, 2 \qquad z_3 = \log m_0 \end{aligned} \tag{4.18}$$

with no world volume flux, $F = 0$, and where m_0 is an arbitrary constant. The first supersymmetry condition (3.1) can be analyzed by keeping the $4, 2, 0$ forms of the pulled back pure spinor Φ_1

$$\begin{aligned} i\Phi_1|_0 &= -\frac{e^A}{8}(\cos^2\phi - \sin^2\phi) \\ i\Phi_1|_2 &= \frac{ie^{-A}}{8}(j + \cos\phi\sin\phi(e^{i(\alpha-\beta)}\omega - e^{-i(\alpha-\beta)}\bar{\omega})) \\ i\Phi_1|_4 &= \frac{e^{-3A}}{16}(\cos^2\phi - \sin^2\phi) j \wedge j \end{aligned}$$

Taking the imaginary part of these expressions we obtain

$$P_\Sigma[\mathrm{Im}(i\Phi_1)] \wedge e^{-P[B]} = -\frac{e^A}{8} P[j] \wedge P[B] = 0 \tag{4.19}$$

This vanishes given the explicit expressions of j (4.5) and B (4.9) and reminding $P_\Sigma[z] = 0$. The only non trivial supersymmetry condition of (3.2) is on the z component. The projection on the pure spinor Φ_2 is

$$\begin{aligned} P_\Sigma[(i_z + g_{z\bar{z}}\bar{z}\wedge)\Phi_2] = \frac{1}{8}\Big(&- ie^{i(\alpha+\beta)}\sin 2\phi + e^{-2A}e^{2i\alpha}\cos^2\phi\,\omega + \\ &+ e^{-2A}e^{2i\beta}\sin^2\phi\,\bar{\omega} + \frac{i}{2}e^{-4A}e^{i(\alpha+\beta)}\sin 2\phi\, j \wedge j\Big) \end{aligned}$$

The pullback of the NS two form (4.9) is

$$\begin{aligned} P_\Sigma[B] = -\frac{\sqrt{\det A}\cos^2\phi}{e^{2A}\sin 2\phi}\Big(&e^{i(\alpha-\beta)}(d\xi_4 + id\xi_6) \wedge (d\xi_5 + id\xi_7) + \\ &+ e^{-i(\alpha-\beta)}(d\xi_4 - id\xi_6) \wedge (d\xi_5 - id\xi_7)\Big) \end{aligned} \tag{4.20}$$

We then compute the terms which contribute to the z component of (3.2)[4]

$$\begin{aligned} P_\Sigma[(i_z + g_{z\bar{z}}\bar{z}\wedge)\Phi_2]|_4 &= \frac{ie^{i(\alpha+\beta)}}{e^{4A}16}\det A\cos\phi\sin\phi\; d\mathrm{Vol}_\Sigma \\ P_\Sigma[(i_z + g_{z\bar{z}}\bar{z}\wedge)\Phi_2]|_2 \wedge (-P_\Sigma[B]) &= \frac{ie^{i(\alpha+\beta)}}{16}\frac{\cos\phi\det A}{e^{4A}\sin\phi}(\cos^2\phi - \sin^2\phi)\, d\mathrm{Vol}_\Sigma \\ P_\Sigma[(i_z + g_{z\bar{z}}\bar{z}\wedge)\Phi_2]|_0 \wedge \frac{1}{2}P_\Sigma[B] \wedge P_\Sigma[B] &= -\frac{ie^{i(\alpha+\beta)}}{16}\frac{\cos^3\phi\det A}{e^{4A}\sin\phi}\, d\mathrm{Vol}_\Sigma \end{aligned}$$

[4]We denote the volume on the wrapped cycle with $d\mathrm{Vol}_\Sigma = (-4d\xi_4 \wedge d\xi_5 \wedge d\xi_6 \wedge d\xi_7)$.

JHEP09(2007)123

Adding these three contributions we conclude that

$$P_\Sigma[(i_z + g_{z\bar{z}}\bar{z}\wedge)\Phi_2] \wedge e^{-P[B]} = 0 \tag{4.21}$$

Then the configuration (4.18) is supersymmetric for the whole family of backgrounds considered in section 4.1, not only the *toric subclass*.

Other flavour embeddings We look also for other $D7$ brane embeddings which preserve supersymmetry in the supersymmetric family of backgrounds of sec 4.1. The computations of the supersymmetry conditions (3.1) and (3.2) are less easy but can be done with the same procedure outlined above. We list the relevant results.

We can place the $D7$ brane orthogonal to one of the other complex coordinates

$$z_k = \log m_0 \qquad z_j = \xi_4 + i\xi_5 \qquad z_3 = \xi_6 + i\xi_7 \qquad k \neq j = 1, 2 \tag{4.22}$$

and after a long computation we find that this is a supersymmetric configuration, satisfying (3.1) and (3.2).

Other possible embeddings are submanifolds like the one suggested in [18], with chiral symmetry breaking. We observe that the complex coordinates we are using (see the appendix A) are the exponential of the usual complex coordinates which are in correspondence with the chiral adjoint fields. Hence we consider embeddings like $e^{z_i}e^{z_j} = m_0^2$. We have to distinguish between two different cases. The first one involves the z_3 component

$$\begin{aligned} e^{z_j}e^{z_3} &= m_0^2 \\ z_k &= \xi_4 + i\xi_5 \qquad z_j = \xi_6 + i\xi_7 \qquad z_3 = \log m_0^2 - (\xi_6 + i\xi_7) \qquad k \neq j = 1, 2 \end{aligned}$$

This configuration turns out to be non supersymmetric.

The second case does not involve the z_3 coordinate

$$\begin{aligned} e^{z_1}e^{z_2} &= m_0^2 \\ z_1 &= \xi_4 + i\xi_5 \qquad z_2 = \log m_0^2 - (\xi_4 + i\xi_5) \qquad z_3 = \xi_6 + i\xi_7 \end{aligned} \tag{4.23}$$

and it results supersymmetric.

The dual flavoured gauge theory. The $D7$ supersymmetric embeddings presented here (4.18), (4.22), (4.23) can be used to add flavours to the PW flow.

If we add N_f $D7$-branes in the configuration (4.18) the dual gauge theory is $\mathcal{N} = 1$ SYM with three chiral adjoint fields and N_f massive flavours with mass m_0, with superpotential

$$W = W_{\mathcal{N}=4} + m\mathrm{Tr}\Phi_3^2 + \mathrm{tr}\ Q\Phi_3\tilde{Q} + m_0\,\mathrm{tr}\ Q\tilde{Q} \tag{4.24}$$

where the first two terms are the mass deformation of $\mathcal{N} = 4$ SYM (4.1). Since we are neglecting the back-reaction of the $D7$ branes, the geometry filled by the $D7$-branes in the IR is warped AdS_5 and the theory flows to the same IR fixed point. For $m_0 \neq 0$, the $D7$-branes end before reaching the IR.

JHEP09(2007)123

If we add N_f $D7$-branes as in (4.22) the gauge theory dual is again $\mathcal{N}=1$ SYM with three chiral adjoint fields and N_f massive flavours, with superpotential

$$W = W_{\mathcal{N}=4} + m\mathrm{Tr}\Phi_3^2 + \mathrm{tr}\ \ Q\Phi_k\tilde{Q} + m_0\,\mathrm{tr}\ Q\tilde{Q} \qquad k=1,2 \tag{4.25}$$

The flavours $Q\tilde{Q}$ now couple to the massless adjoint field Φ_k.

Finally, if we add N_f $D7$-branes embedded as (4.23) the dual flavoured gauge theory is $\mathcal{N}=1$ SYM with three chiral adjoint fields and two different N_f massive flavours, with superpotential

$$W = W_{\mathcal{N}=4} + m\mathrm{Tr}\Phi_3^2 + \mathrm{tr}\ \ Q_1\Phi_1\tilde{Q}_1 + \mathrm{tr}\ \ Q_2\Phi_2\tilde{Q}_2 + m_0\,\mathrm{tr}\ (Q_1\tilde{Q}_2 + Q_2\tilde{Q}_1) \tag{4.26}$$

where Q_1 and Q_2 denote the two flavours. This configuration can be interpreted as two sets of N_f $D7$-branes at $e^{z_1} = m_0$ and $e^{z_2} = m_0$ respectively, each supporting different flavours, which are joint smoothly into one set of N_f $D7$ branes wrapped on $e^{z_1}e^{z_2} = m_0^2$ [18]. On the dual gauge theory picture there are two flavour groups $\mathrm{SU}(N_f)_1 \times \mathrm{SU}(N_f)_2$ broken to the diagonal subgroup by the mass term m_0.

4.4 Effective Strings

We take D-branes that fill two coordinates in the Minkowski space time, for example at $x_2 = x_3 = 0$, filling $\xi_0 = x_0, \xi_1 = x_1$. They can be viewed as propagating strings in the four dimensional description. However, when the wrapped cycle of the internal manifold is non compact, the effective string tension in the four dimensional picture diverges. The supersymmetry conditions are the pair (3.1) and (3.2) in the ES case of table 1. We find supersymmetric embeddings of both $D3$ and $D7$ branes which involve non compact cycles in the internal manifold. The $D3$ brane wraps a two cycle, whereas the $D7$ brane fills the whole internal manifold. Our analysis concern the whole family of backgrounds presented in section 4.1.

$D3$ effective strings. We place $D3$-brane probes filling two directions in the internal space. We fix the z_3 coordinate, i.e. $z_3 = c_3 e^{i\tau_3}$ and we look for supersymmetric embeddings filling z_1 and z_2. The embedding along the two complex coordinates, $z_k = e^{i\tau_k}(\xi_{k+1} + ic_k)$ for $k = 1,2$ results non supersymmetric.

On the other hand, the non compact embedding where we identify z_1 and z_2 except for constant phases and shifts

$$z_1 = e^{i\tau_1}(\xi_2 + c_1 + i(\xi_3 + c_2)) \qquad z_2 = e^{i\tau_2}(\xi_2 - c_1 + i(\xi_3 - c_2)) \qquad z_3 = c_3 e^{i\tau_3} \tag{4.27}$$

results supersymmetric for any choice of the phases τ_k and of the real constants c_k.

$D7$ effective strings. We probe the geometry with $D7$-brane covering the whole internal space

$$z_k = \xi_{k+1} + i\xi_{k+4} \qquad k = 1, \dots, 3 \tag{4.28}$$

By a long but straightforward computation we find that this is a supersymmetric embedding, which satisfies the supersymmetry conditions.

JHEP09(2007)123

5. *D*-branes on the beta deformed background

5.1 Beta deformation of $\mathcal{N} = 4$ SYM and its gravity dual

The $\mathcal{N} = 1$ beta deformed gauge theory is a marginal deformation [22] of the $\mathcal{N} = 4$ SYM, with superpotential

$$W_\beta = h\,Tr(e^{i\pi\beta}\Phi_1\Phi_2\Phi_3 - e^{-i\pi\beta}\Phi_1\Phi_3\Phi_2) \tag{5.1}$$

where Φ_i are the three chiral adjoint superfields, and β a complex constant. We consider β to be real; in this case it is usually denoted as γ. Besides the $\mathrm{U}(1)_R$ symmetry, this theory has two global symmetries $\mathrm{U}(1)_a \times \mathrm{U}(1)_b$ with charges

	Φ_1	Φ_2	Φ_3
$\mathrm{U}(1)_a$	0	1	-1
$\mathrm{U}(1)_b$	-1	1	0

These two global symmetries were crucial in the generating solutions technique of [13], where the supergravity background dual to such gauge theory has been obtained. This background has been analyzed using generalized complex geometry in [11]. The ten dimensional metric is

$$ds^2 = e^{2A}ds^2_{\mathrm{Mink}} + ds^2_6, \qquad ds^2_6 = e^{-2A}d\tilde{s}^2_6 \tag{5.2}$$

where $\tilde{ds}^2_6$ is the rescaled internal metric. The internal SU(2) structure manifold can be described by local complex coordinates

$$\begin{aligned} z_1 &= r\mu_1 e^{i\sigma_1} = r\cos\alpha e^{i(\psi-\varphi_2)} \\ z_2 &= r\mu_2 e^{i\sigma_2} = r\sin\alpha\cos\theta e^{i(\psi+\varphi_1+\varphi_2)} \\ z_3 &= r\mu_3 e^{i\sigma_3} = r\sin\alpha\sin\theta e^{i(\psi-\varphi_1)} \end{aligned} \tag{5.3}$$

The almost complex structure can be expressed [11] in terms of 1-forms (for details see the appendix B) which give the rescaled metric a simple expression

$$d\tilde{s}^2_6 = x_1^2 + x_2^2 + G(y_1^2 + y_2^2) + z\bar{z} \tag{5.4}$$

where

$$G = \frac{1}{1+\gamma^2 g} \qquad z = \frac{d(z_1z_2z_3)}{r^2\sqrt{g}} \qquad g = \mu_1^2\mu_2^2 + \mu_2^2\mu_3^2 + \mu_3^2\mu_1^2 \qquad e^{2A} = r^2 \tag{5.5}$$

The background has non trivial dilaton, RR and NS fluxes

$$e^\phi = \sqrt{G} \tag{5.6}$$

$$B_2 = \gamma\sqrt{g}G\frac{y_1\wedge y_2}{r^2} \tag{5.7}$$

$$F_3 = 12\gamma\cos\alpha\sin^3\alpha\sin\theta\cos\theta d\psi\wedge d\alpha\wedge d\theta \tag{5.8}$$

$$F_5 = 4(\mathrm{vol}_{AdS_5} + *\mathrm{vol}_{AdS_5}) \tag{5.9}$$

This solution differs from the family of backgrounds reviewed in section 4.1, for example the dilaton is not constant here. However it is an SU(2) structure manifold which can be described by the ansatz (2.13) and (2.14) for the spinors [11]. The 1-form z in (5.4) is a globally defined vector. The 2-forms j and ω are

$$j = \sqrt{G}(x_1 \wedge y_1 + x_2 \wedge y_2) \tag{5.10}$$

$$\omega = i(x_1 + i\sqrt{G}y_1) \wedge (x_2 + i\sqrt{G}y_2) \tag{5.11}$$

and

$$a = ix = ie^{A/2}\cos\phi = \frac{i}{\sqrt{2}}e^{A/2}(1+\sqrt{G})^{\frac{1}{2}} \tag{5.12}$$

$$b = -iy = -ie^{A/2}\sin\phi = \frac{i}{\sqrt{2}}e^{A/2}(1-\sqrt{G})^{\frac{1}{2}} \tag{5.13}$$

The phases α and β in (2.14) are vanishing, $\alpha = \beta = 0$. Once again the pure spinors (2.12) are constructed with the rescaled forms $(j, \omega) \to (e^{-2A}j, e^{-2A}\omega)$ and $z \to e^{-A}z$ which refer to the complete six dimensional metric (5.2).

We look for supersymmetric embeddings of D-branes in this background employing the conditions (3.1) and (3.2).

5.2 $D5$ domain walls

We look for $D5$-brane embeddings filling three directions in the internal manifold and placed in Minkowski at $x_3 = 0$ with ($\xi_\mu = x_\mu$, $\mu = 0, 1, 2$). We choose the following ansatz, which is supersymmetric in the undeformed $\gamma = 0$ case ($AdS_5 \times S^5$),

$$z_k = e^{-i\tau_k}(\xi_{k+2} + ic_k) \qquad \bar{z}_k = e^{i\tau_k}(\xi_{k+2} - ic_k) \qquad k = 1, \ldots, 3 \tag{5.14}$$

where τ_k, c_k are arbitrary real constants. Computing the supersymmetry conditions (3.1) and (3.2)[5] this embedding results non supersymmetric for any choice of the constants τ_k, c_k. For instance in the simple case ($\tau_k = 0, c_k = 0$) the z and $\bar{z}$ components of the supersymmetry conditions (3.2) can be computed

$$\frac{1}{3}P_\Sigma[(g^{\bar{z}z}i_z + \bar{z}\wedge)\Phi_2] \wedge e^{-P[B]} = P_\Sigma[(g^{z\bar{z}}i_{\bar{z}} + z\wedge)\Phi_2] \wedge e^{-P[B]} = -\frac{i}{16}e^{-A}\gamma\sqrt{g\,G} \tag{5.15}$$

where the functions (A, g, G) are intended evaluated on the world volume. The result (5.15) cannot vanish unless $\gamma = 0$, i.e. the undeformed case; hence the embedding (5.14) is not supersymmetric in the beta deformed background.

5.3 $D7$ flavour branes

We look for supersymmetric $D7$ configurations filling the Minkowski space time $\xi_\mu = x_\mu$ ($\mu = 0, \ldots, 3$) and wrapped on a non compact four cycle in the internal manifold, suitable for adding flavour to the beta deformed theory. As already observed, an SU(2) structure manifold is characterized by a globally defined vector (z), and a natural four cycle Σ is

[5] In the DW case of table 1.

	Φ_1	Φ_2	Φ_3	Q_1	$\tilde{Q}_1$	Q_2	$\tilde{Q}_2$	Q_3	$\tilde{Q}_3$
$\mathrm{U}(1)_a$	0	1	-1	1	-1	0	-1	1	0
$\mathrm{U}(1)_b$	-1	1	0	0	1	-1	0	-1	1

Table 2: $\mathrm{U}(1)_a \times \mathrm{U}(1)_b$ charges of chiral superfields.

where $P_\Sigma[z] = 0$. In the beta deformed background the vector z is (5.5), and the condition $P_\Sigma[z] = 0$ implies, in complex coordinates,

$$z_1 z_2 z_3 = m^3 \tag{5.16}$$

with m constant.

We then take the following four cycle embedding for $D7$-branes

$$z_k = \xi_{k+3} e^{i\xi_{k+5}} \qquad k = 1, 2 \qquad\qquad z_3 = \frac{m^3}{\xi_4 e^{i\xi_6} \xi_5 e^{i\xi_7}} \tag{5.17}$$

with no world volume flux, i.e. $F = 0$. By direct inspection we find that this embedding satisfies the conditions[6] (3.1) and (3.2), and hence is supersymmetric. It preserves the translational invariance of φ_1 and φ_2. We then expect the $\mathrm{U}(1)_a$ and $\mathrm{U}(1)_b$ symmetries to be preserved in the dual gauge theory description.

This embedding and the dual flavoured gauge theory can be explained as follows. We have three sets of N_f $D7$ branes located at $z_1 = m$, $z_2 = m$, $z_3 = m$ respectively, each one supporting a flavour group $\mathrm{SU}(N_f)$. We can join these branes à la Karch and Katz [18] and obtain one single set of N_f $D7$ branes located as in (5.17). These $D7$-branes terminate before reaching the IR region and the conformal invariance is explicitly broken by the mass m, which also breaks the flavour groups $\mathrm{SU}(N_f) \times \mathrm{SU}(N_f) \times \mathrm{SU}(N_f)$ to the diagonal subgroup.

In order to deduce the superpotential of the dual gauge theory we observe that the same configuration can be realized in the undeformed ($\gamma = 0$, $AdS_5 \times S^5$) case; here the superpotential is the following[7]

$$W = W_{N=4} + \mathrm{tr}\ Q_1 \Phi_1 \tilde{Q}_1 + \mathrm{tr}\ Q_2 \Phi_2 \tilde{Q}_2 + \mathrm{tr}\ Q_3 \Phi_3 \tilde{Q}_3 + m\, \mathrm{tr}\ (Q_1 \tilde{Q}_2 + Q_2 \tilde{Q}_3 + Q_3 \tilde{Q}_1) \tag{5.18}$$

Note that the massive flavours preserves the $\mathrm{U}(1)_a \times \mathrm{U}(1)_b$ symmetry, assigning the charges as in table 2. Now, for N_f $D7$ branes embedded as (5.17) in the beta deformed background, the dual gauge theory is beta deformed $\mathcal{N} = 1$ SYM coupled to three different massive flavours. The resulting phase factors of the terms in the superpotential (5.18) can be easily obtained following the prescription of [13] with the charges in table 2, having

$$\begin{aligned} W \;=\; & W_{\beta=\gamma} + e^{-i\pi\gamma}\, \mathrm{tr}\ Q_1 \Phi_1 \tilde{Q}_1 + e^{i\pi\gamma}\, \mathrm{tr}\ Q_2 \Phi_2 \tilde{Q}_2 + e^{-i\pi\gamma}\, \mathrm{tr}\ Q_3 \Phi_3 \tilde{Q}_3 + \\ & + m\, \mathrm{tr}\ (Q_1 \tilde{Q}_2 + Q_2 \tilde{Q}_3 + Q_3 \tilde{Q}_1) \end{aligned} \tag{5.19}$$

Note that the flavour mass terms are not affected by the beta deformation.

[6] In the STF case of table 1.

[7] We set the couplings to one for simplicity.

JHEP09(2007)123

Other $D7$ embeddings. If we do not require the $\mathrm{U}(1)_a$ and $\mathrm{U}(1)_b$ global symmetries to be preserved we can try to embed the $D7$ branes in other submanifolds, with vanishing world volume flux. The computations of the supersymmetry conditions (3.1) and (3.2) get more complicated.

We take the embeddings

$$\begin{aligned} &\xi_\mu = x_\mu \qquad \mu = 0,\dots,3 \\ &z_i = \xi_4 e^{i\xi_6} \qquad z_j = \xi_5 e^{i\xi_7} \qquad z_k = m_0 \qquad i \neq j \neq k = 1,2,3 \end{aligned} \tag{5.20}$$

A long computation shows they are supersymmetric for any choice of the mass m_0. Here the dual gauge theory is beta deformed $\mathcal{N}=1$ SYM plus N_f flavours[8] which couple with the adjoint field Φ_k.

Finally, after a long computation, we find that the following $D7$ embeddings with chiral symmetry breaking are supersymmetric

$$\begin{aligned} &\xi_\mu = x_\mu \qquad \mu = 0,\dots,3 \\ &z_i = \xi_4 e^{i\xi_6} \qquad z_j = \xi_5 e^{i\xi_7} \qquad z_k = \frac{m_0^2}{\xi_5 e^{i\xi_7}} \qquad i \neq j \neq k = 1,2,3 \end{aligned} \tag{5.21}$$

The dual gauge theory is beta deformed $\mathcal{N}=1$ SYM with two kinds of N_f massive flavours Q_1 and Q_2, which couple to Φ_j and Φ_k, respectively. The mass m_0 breaks the flavour groups $\mathrm{SU}(N_f)_1 \times \mathrm{SU}(N_f)_2$ to the diagonal subgroup.

For these additional $D7$ embeddings the superpotential terms and their phase factors can be obtained with the same procedure followed in the derivation of (5.19), by starting from the $\mathcal{N}=4$ case (i.e. $\gamma=0$).

5.4 Effective strings

Finally we take D-branes that fill just two coordinates in the Minkowski space time ($\xi_0 = x_0, \xi_1 = x_1$). We place them at $x_2 = x_3 = 0$. We do not find supersymmetric configurations of $D3$ or $D5$ branes. We instead find that a $D7$-brane covering the whole internal space

$$z_k = \xi_{k+1} + i\xi_{k+4} \qquad k = 1,\dots,3 \tag{5.22}$$

is supersymmetric.

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[8] N_f is the number of $D7$ branes.

A. The supersymmetric family of backgrounds and IR PW

The supersymmetry equations for the ansatz (2.13), (2.14) was studied in [11]. They imply, for complex solutions with constant dilaton, that the geometrical quantities can be expressed as derivatives of a single function F. If the background does not depend on σ_3 we have

$$A_{i\bar{j}} = \frac{\partial^2 F}{\partial z_i \partial \bar{z}_j} \quad i,j = 1,2 \tag{A.1}$$

$$A_{i\bar{j}}\bar{\alpha}_j = \frac{\partial^2 F}{\partial z_i \partial \bar{z}_3}\,, \tag{A.2}$$

$$\alpha_i A_{i\bar{j}} = \frac{\partial^2 F}{\partial \bar{z}_j \partial z_3}\,, \tag{A.3}$$

$$u^2 a_3 \cos 2\phi + \alpha_i A_{i\bar{j}}\bar{\alpha}_j = \frac{\partial^2 F}{\partial z_3 \partial \bar{z}_3}\,. \tag{A.4}$$

$$a_3 u^2 \sin^2\phi = -\frac{\partial}{\partial z_3} F\,. \tag{A.5}$$

The infrared geometry of the PW flow can be reconstructed in this family of supersymmetric backgrounds as follows [11]. Choose coordinates

$$\begin{aligned} e^{z_1} &= r^{3/4}\cos\theta\cos\varphi e^{i\sigma_1}\,, \\ e^{z_2} &= r^{3/4}\cos\theta\sin\varphi e^{i\sigma_2}\,, \\ e^{z_3} &= r^{3/2}\sin\theta e^{i\sigma_3}\,. \end{aligned}$$

The generalized Kahler potential F is

$$F = \frac{3}{4} r^2 (1 - 2\sin^2\theta)\,, \tag{A.6}$$

and the warp factor

$$e^{2A} = r^2 \sqrt{\frac{3}{4}(1+\sin^2\theta)} \tag{A.7}$$

The other quantities are determined, for example

$$\sin 2\phi = \frac{\sin\theta\sqrt{2+\sin^2\theta}}{1+\sin^2\theta} \tag{A.8}$$

$$A_{1\bar{1}} = r^2\left(\cos^2\theta\cos^2\varphi + \frac{\cos^4\theta\cos^4\varphi}{3+3\sin^2\theta}\right) \tag{A.9}$$

$$A_{1\bar{2}} = A_{2\bar{1}} = \frac{r^2\cos^4\theta\sin^2\varphi\cos^2\varphi}{3+3\sin^2\theta} \tag{A.10}$$

$$A_{2\bar{2}} = r^2\left(\cos^2\theta\sin^2\varphi + \frac{\cos^4\theta\sin^4\varphi}{3+3\sin^2\theta}\right) \tag{A.11}$$

$$a_3 = \frac{1+\sin^2\theta}{4r(2+\sin^2\theta)} \tag{A.12}$$

B. Beta deformed gravity dual

We have already introduced the complex coordinates z_i (5.3); the one forms appearing in (5.4) are defined as [11]

$$x_1 + iy_1 = e^{-i\sigma_1}\sqrt{\frac{g}{\mu_1^2(\mu_2^2+\mu_3^2)}}\left(dz_1 - \frac{\bar{z}_2\bar{z}_3 z}{r^2\sqrt{g}}\right) \tag{B.1}$$

$$x_2 + iy_2 = e^{-i\sigma_2}\sqrt{1+\frac{\mu_3^2}{\mu_2^2}}\left(dz_2 - \frac{\bar{z}_1\bar{z}_3 z}{r^2\sqrt{g}}\right) + \frac{\mu_3^2 e^{-i\sigma_1}}{\mu_1\sqrt{\mu_2^2+\mu_3^2}}\left(dz_1 - \frac{\bar{z}_2\bar{z}_3 z}{r^2\sqrt{g}}\right) \tag{B.2}$$

$$z = \frac{d[z_1 z_2 z_3]}{r^2\sqrt{g}} \tag{B.3}$$

The internal metric (5.4) gives then [13]

$$d\tilde{s}_6^2 = dr^2 + r^2\left(\sum_{i=1}^{3}(d\mu_i^2 + G\mu_i^2 d\sigma_i^2) + \gamma^2 G\mu_1^2\mu_2^2\mu_3^2(d\sigma_1+d\sigma_2+d\sigma_3)^2\right) \tag{B.4}$$

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JHEP09(2007)123